1. Let $\mathbf{F}(x, y, z) = (x^3 \ln z)\mathbf{i} + (xe^{-y})\mathbf{j} - (y^2 + 2z)\mathbf{k}$. Calculate the divergence of $\mathbf{F}$ at the point (2, ln 2, 1).
   a. -3
   b. 2
   c. $\frac{1}{2}$
   d. $-2 \ln 2\mathbf{i} + 8\mathbf{j} + \frac{1}{2}\mathbf{k}$
   e. None of these

2. Let $\mathbf{F}(x, y, z) = \cos x\mathbf{i} + \sin y\mathbf{j} + z\mathbf{k}$. Calculate the divergence of $\mathbf{F}$ at the point $\left(\frac{\pi}{2}, \pi, 1\right)$.
   a. $-\mathbf{i} - \mathbf{j} + \mathbf{k}$
   b. 1
   c. -1
   d. None of these
   e. 0

3. Let $\mathbf{F}(x, y, z) = \cos x\mathbf{i} + \sin y\mathbf{j} + e^{xy}\mathbf{k}$. Calculate **curl F** at the point (1, 1, 1).
   a. cos 1 - sin 1
   b. $e\mathbf{i} - e\mathbf{j}$
   c. None of these
   d. $-\frac{\pi}{2}$
   e. $e\mathbf{i} + e\mathbf{j}$

4. Let $\mathbf{F}(x, y, z) = (x^3 \ln z)\mathbf{i} + xe^{-y}\mathbf{j} - (y^2 + 2z)\mathbf{k}$. Calculate **curl F** at (1, 1, 1).
   a. $-2\mathbf{i} + \mathbf{j} + \frac{1}{e}\mathbf{k}$
   b. None of these
   c. $-2\mathbf{i} - \mathbf{j} + \frac{1}{e}\mathbf{k}$
   d. -3
   e. $\frac{-1 - 2e}{e}$

5. Let $\mathbf{F}(x, y, z) = (2xy + z^2)\mathbf{i} + x^2\mathbf{j} + (2xz + \pi \cos \pi z)\mathbf{k}$. Find the potential function of **F**.
a. $x^2y + xz^2 - \sin \pi z + C$
b. None of these
c. $2x^2y + 2xz^2 + \sin \pi z + C$
d. $x^2y + xz^2 + \sin \pi z + C$
e. $2x^2y + 2xz^2 - \pi \sin \pi z + C$

6. Let $\mathbf{F}(x, y) = e^y\mathbf{i} + (xe^y + y)\mathbf{j}$. Find the potential function of **F**.
a. $2xe^y + \frac{y^2}{2} + C$
b. None of these
c. $xe^y + \frac{y^2}{2} + C$
d. $xe^y + 1 + C$
e. $4xe^y + y^2 + C$

7. Find the divergence of the vector field
$\mathbf{F}(x, y, z) = \cos(xy)\mathbf{i} - \sin(xz)\mathbf{k}$ at $\left(1, \frac{\pi}{2}, 0\right)$.
a. $-\frac{\pi}{2}$
b. $\frac{\pi}{2} - 1$
c. $-\frac{\pi}{2} + 1$
d. None of these
e. -1

8. Determine which of the following fields is conservative.
a. $(4x^2 - 4y^2)\mathbf{i} + (8xy - \ln y)\mathbf{j}$
b. None of these
c. $(8xy - \ln y)\mathbf{i} + (4y^2 - 4x^2)\mathbf{j}$
d. $(2xy^3 + x + z)\mathbf{i} + (3x^2y^2 - y)\mathbf{i} + (y + \sin z)\mathbf{k}$
e. $e^y\mathbf{i} + (xe^y + y)\mathbf{j}$

9. Find the curl of $\mathbf{F}(x, y, z) = xy\mathbf{i} - z\mathbf{j} + x\mathbf{k}$ at (1, 0, 1).
a. $\mathbf{j} + \mathbf{k}$
b. None of these
c. $\mathbf{i} - \mathbf{j} - \mathbf{k}$
d. $-\mathbf{j} - \mathbf{k}$
e. $\mathbf{j} - \mathbf{k}$

10. Let C be the line segment from the point (0, 0, 0) to the point (1, -3, 2). Find $\int_C (x + y^2 - 2z)\,ds$.

a. None of these
b. $\frac{3}{2}$
c. $\frac{3}{2}\sqrt{14}$
d. $-\frac{25}{2}$
e. 10

11. Let C be the line segment from the point (0, 0, 0) to the point (1, 3, -2). Find $\int_C (x + y^2 - 2z)\,ds$.

a. None of these
b. $\frac{3}{2}$
c. $\frac{3}{2}\sqrt{14}$
d. $\frac{11}{2}$
e. $\frac{11}{2}\sqrt{14}$

12. Let $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$, and evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the curve $\mathbf{r}(t) = t\mathbf{i} + \cos t\mathbf{j} + \sin t\mathbf{k}$, $0 \le t \le 2\pi$.

a. $1 - \pi$
b. $\pi - 1$
c. 2π
d. None of these
e. -2π

13. Evaluate the line integral of $\mathbf{F}(x, y) = xy\mathbf{i} - x^2\mathbf{j}$ where C is the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$, $0 \le t \le 2$.

a. -1
b. 4
c. -4
d. $-\frac{1}{4}$
e. None of these

14. Evaluate $\int_C (x + 2y)\,ds$ where C is the path counterclockwise around the upper half of the unit circle.

a. -4
b. None of these
c. 4
d. 0
e. -2

15. Evaluate $\int_C (x^3y + 1)\,ds$ if C is the path counterclockwise around the upper half of the unit circle.

a. $-\frac{1}{2} + \pi$
b. $\frac{1}{2} + \pi$
c. None of these
d. $-\frac{1}{4} + \pi$
e. $\pi - 1$

16. For the force field $\mathbf{F}(x, y, z) = y \ln z\mathbf{i} + x \ln z\mathbf{j} + \frac{xy}{z}\mathbf{k}$, calculate the work done by $\mathbf{F}$ on an object moving along a curve from $(1, 1, e)$ to $(1, 2, e^2)$.

a. 2
b. 0
c. None of these
d. 3
e. 1

17. Use the Fundamental Theorem of Line Integrals to evaluate
$$\int_C (y^2 - 3x^2)\,dx + (2xy + 2)\,dy,$$ where C is a smooth curve from (1, 1) to (-1, 0).

a. 1
b. -1
c. None of these
d. 0
e. -2

18. Evaluate $\iint_S z\,dS$, where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$.

a. None of these
b. $18\pi\sqrt{2}$
c. $\dfrac{52\pi}{\sqrt{2}}$
d. $2\sqrt{2}\pi$
e. $\dfrac{26\pi\sqrt{2}}{3}$

19. Let Q be the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes and the planes $z = 1$. Use the Divergence Theorem to calculate
$\iint_S \mathbf{F} \cdot \mathbf{N}\,dS$ where S is the surface of Q and
$\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

a. None of these
b. 6π
c. 1
d. 0
e. 3π

20. Let S be the surface bounded by the xy-plane and the top hemisphere of $x^2 + y^2 + z^2 = 4$. Let $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$. Which of the following integrals uses the Divergence Theorem to calculate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$?

a. $\iint_S (3x^2 + 3y^2 + 3z^2)\, ds$

b. $\iiint_Q (x^3 + y^3 + z^3)\, dV$, where Q is the solid with surface S

c. None of these

d. $\iint_S (x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k})$ **Á** $(\mathbf{i} + \mathbf{j} + \mathbf{k})\, ds$

e. $\iiint_Q (3x^2 + 3y^2 + 3z^2)\, dV$, where Q is the solid with surface S

21. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$, if S is the cube bounded by the plane $x = \pm 1$, $y = \pm 1$, $z = \pm 1$, and $\mathbf{F} = x\mathbf{i}$.

a. 0
b. 8
c. 16
d. None of these
e. 4

22. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$, if S is the cube bounded by the plane $x = \pm 1$, $y = \pm 1$, $z = \pm 1$, and $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$.

a. 16
b. None of these
c. 0
d. 8
e. 4

23. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$ if $\mathbf{F} = x(y + 1)\mathbf{i} + 2yz\mathbf{j} - (z^2 + yz)\mathbf{k}$ and S is the sphere $x^2 + y^2 + z^2 = 1$.

a. $\frac{2}{3}\pi$
b. 2π
c. None of these
d. $\frac{\pi}{3}$
e. 4π

24. Let C be the triangle oriented counterclockwise lying in the plane $x + 2y + z = 6$. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = z\mathbf{i} - x^2\mathbf{j} + y\mathbf{k}$.

a. -54
b. None of these
c. -9
d. 11
e. 19

25. Let C be the triangle from (0, 0, 0) to (2, 0, 0) to (0, 2, 1) to (0, 0, 0) which lies in the plane $z = \frac{y}{2}$. If $\mathbf{F}(x, y, z) = -3y^2\mathbf{i} + 4z\mathbf{j} + 6x\mathbf{k}$, calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Stokes's Theorem.

a. 2
b. 0
c. $\frac{2}{3}$
d. None of these
e. 14

26. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = z\mathbf{i} + 2x\mathbf{j} + 2y\mathbf{k}$. S is the paraboloid (oriented upward) $z = 4 - x^2 - y^2$, $z \geq 0$, and C is its boundary.

a. None of these
b. 8π
c. 4π
d. 16π
e. 12π

27. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$. S is the paraboloid (oriented upward) $z = x^2 + y^2$, $0 \le z \le 1$, and C is its boundary.

a. None of these
b. 2π
c. $-\pi$
d. π
e. 0

28. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = (3z - 2y)\mathbf{i} + (4x - 3y)\mathbf{j} + (z + 2y)\mathbf{k}$ and C is the boundary of the triangle joining the points (1, 0, 0), (0, 1, 0), and (0, 0, 1).

a. $\frac{11}{4}$
b. None of these
c. $\frac{11}{\sqrt{3}}$
d. $\frac{11}{2}$
e. $\frac{11\sqrt{3}}{4}$

29. Use Stokes's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = 2z\mathbf{i} - x\mathbf{j} + 3y\mathbf{k}$, and S is the portion of the plane (oriented upward) $3x + 3y + 2z = 6$ in the first octant and C is its boundary.

a. 14
b. 18
c. None of these
d. 13
e. 16

30. Evaluate the surface integral $\iint_S z\, dS$ where S is the hemisphere $z = \sqrt{16 - x^2 - y^2}$.

a. None of these
b. 4π
c. 64π
d. 32π
e. 8π

31. Use Green's Theorem to evaluate the line integral $\int_C (x^4 - 3y)\,dx + (y^4 + 5x)\,dy$, where C is the circle of radius 2, $x^2 + y^2 = 4$.

a. 6π
b. 12π
c. 18π
d. None of these
e. 24π

32. Which of the following integrals calculates the area of the region bounded by $y = 4x^2$ and $y = 16x$?

a. $\int_0^4 (4x^2 - 16x)\,dx$
b. $\frac{1}{2}\int_0^4 4x^2\,dx$
c. $\int_0^{64} (4x^2 - 16x)\,dy$
d. $\int_0^4 \left[\frac{1}{2}\sqrt{y} - \frac{y}{16}\right] dy$
e. None of these

33. Use Green's Theorem to evaluate the line integral $\int_C y^2\,dx + 6xy\,dy$, where C is the path from (0, 0) to (1, 0) along $y = 0$, from (1, 0) to (1, 1) along $x = 1$, and from (1, 1) to (0, 0) along $y = \sqrt{x}$.

a. 2
b. None of these
c. 4
d. 1
e. 0

34. Use Green's Theorem to compute $\int_C 2y^3\,dx - x^2\,dy$ where C is the square with vertices (1, 1), (1, -1), (-1, -1) and (-1, 1).

a. 6
b. 4
c. None of these
d. 12
e. 8

35. Let C be the square path from (0, 0) to (1, 0) to (1, 1) to (0, 1) and back to (0, 0). Then $\int_C 5y\,dx - 8x\,dy =$ ____________.

a. None of these
b. $\int_0^1 \int_0^1 (-8 - 5)\,dx\,dy$
c. $\int_0^1 \int_0^1 5 - (-8)\,dx\,dy$
d. $\int_0^1 \int_0^1 0\,dx\,dy$
e. $\int_0^1 \int_0^1 (-8x - 5y)\,dx\,dy$

36. Use Green's Theorem to evaluate the line integral $\int_C (x^2 + 2y)\,dx + \left[\frac{1}{2}x^2 - y^3\right]dy$, where C is the path from (0, 0) to (1, 0) along $y = 0$, from (1, 0) to (1, 1) along $x = 1$ and from (1, 1) to (0, 0) along $y = \sqrt{x}$.

a. $-\frac{6}{5}$
b. $-\frac{14}{15}$
c. $-\frac{4}{15}$
d. None of these
e. $-\frac{2}{3}$

1. a
2. c
3. b
4. a
5. d
6. c
7. d
8. e
9. c
10. c
11. e
12. c
13. c
14. c
15. c
16. d
17. b
18. a
19. e
20. e
21. b
22. c
23. c
24. c
25. e
26. b
27. d
28. d
29. d
30. c
31. d
32. b
33. d
34. e
35. b
36. b